İZMİR INSTITUTE OF TECHNOLOGY
1992

II. ANATOLIAN ALGEBRA WORKSHOP

ABSTRACT BOOK

$$\begin{array}{ccccccccc} 0 & \longrightarrow & A & \xrightarrow{p} & B & \xrightarrow{q} & C & \longrightarrow & 0 \\ & & \downarrow g & & \downarrow f & & \downarrow h & & \\ 0 & \longrightarrow & A' & \xrightarrow[s]{} & B' & \xrightarrow[t]{} & C' & \longrightarrow & 0 \end{array}$$

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2nd Anatolian Algebra Workshop AAW II - İZMİR 2026

BOOK OF ABSTRACTS

July 8-10, 2026
İzmir, Türkiye

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Preface

The 2nd Anatolian Algebra Workshop (AAW II - İZMİR 2026) will be held at İzmir Institute of Technology on July 8-10, 2026. The program includes invited talks and contributed presentations that have undergone a peer-review process.

The aim of the workshop is to bring together researchers working in algebra and its applications, to share recent developments, discuss new ideas, and provide a scientific environment that fosters potential collaborations.

The workshop is designed especially to give young researchers and graduate students the opportunity to present their work, interact with experienced academics, and expand their scientific networks. In this way, it aims to bring together researchers from different generations and encourage the formation of new collaborations. We believe that these workshops are not only serve as a platform for sharing current research but also open new horizons for future studies and contribute to the development of the algebra community.

We hope that this workshop will provide a productive and inspiring scientific environment, foster new collaborations, and be an enjoyable experience for all participants.

Prof. Dr. Engin Büyükaşık
Organizing Committee Chair of AAW II - İZMİR 2026

Dedication

The 2^{nd} Anatolian Algebra Workshop (AAW II - İZMİR 2026) is dedicated to Prof. Rafail Alizade, a distinguished scientist who, through his valuable contributions at every level of mathematics during more than thirty years in our country, has established a lasting academic tradition.

Prof. Alizade has mentored and supervised numerous graduate students, sharing his knowledge and experience with new generations and serving as a guide throughout their academic journeys.

In addition, through his devoted and high quality educational efforts within the framework of mathematical olympiads, he has played a leading role in discovering the talents of many young people and has contributed to their development, enabling them to achieve successes that have brought pride to our country on the international stage.

On this occasion, we proudly express our deepest gratitude, appreciation, and respect to Prof. Rafail Alizade for his outstanding contributions and enduring impact on mathematics and future generations of mathematicians.

Acknowledgements

We extend our sincere appreciation to the members of the Scientific Committee, the Organizing Committee, and the Local Organizing Committee for their dedication, hard work, and valuable contributions.

We are also deeply grateful to the Rectorate of the İzmir Institute of Technology for its support of this workshop, to The Scientific and Technological Research Council of Türkiye (TÜBİTAK) for its support under the 2224-B Program, and to the Turkish Mathematical Society (TMD) for its support through the Mathematics Research Friendly (MAD) Program.

Contents

Abstracts of Invited Speakers

On the Characterization of Minimal Linear Codes

MURAT ALTUNBULAK

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Minimal linear codes are error-correcting codes in which the support of every nonzero codeword does not properly contain the support of another linearly independent codeword. Because of their natural property of having minimal Hamming weight supports, minimal codes have important applications in cryptography, secret sharing schemes and secure multi-party computations. Over the last decade, minimal linear codes have attracted considerable attention due to their rich algebraic structure and their fundamental role in secret sharing schemes and related cryptographic applications.

In this talk we begin with a brief introduction to the fundamental concepts of coding theory and the notion of minimality in linear codes. We will then discuss recent developments in the theory of minimal codes, including new characterization methods, construction techniques, and advances concerning minimal code parameters. Finally, we will present a construction of minimal codes using permutation automorphism groups, which is a part of a joint work with Fatma A. Aksu.

MSC 2020: 94B05, 20B25, 94B60, 11T71

Keywords: Minimal codes, linear codes, permutation automorphism groups

A Long-Standing Open Problem and Automorphism Groups of Linear Codes.

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The existence of an extremal self-dual binary linear code of length 72 is a longstanding open problem in coding theory, first explicitly stated by Neil J. A. Sloane in 1973 (see [5]). One of the main approaches to solving this problem is to investigate the permutation automorphism group of such a code, assuming it exists.

This line of research began with the work of John Horton Conway and Vera Pless in 1982 (see [4]). Since then, a series of papers has investigated the permutation automorphism group of a putative extremal self-dual code of length 72, successfully excluding many subgroups of the symmetric group S_{72}. Building on these results, Martino Borello proved in 2014 that the automorphism group of a putative extremal self-dual binary code has order at most 5 (see [3]). Thus, if such a code exists, its automorphism group must be quite small.

In this talk, we will give a general overview of the problem and present our recent contributions concerning the cases where the automorphism group has order 2 or 3. This is ongoing joint work with Murat Altunbulak, Rogaeh Hafezieh, and İpek Tuvay (see [1, 2]).

MSC 2010: 94B05, 11T71, 20B05

Keywords: Binary self dual code, permutation automorphism group, symmetric groups

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Polyomino Ideals and Rook Polynomials

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Polyominoes are plane figures formed by joining unit squares edge to edge. Given a polyomino P, one can associate a quadratic binomial ideal I_P, called the *polyomino ideal*, generated by certain 2-minors of a matrix of indeterminates. These ideals form a broad class of binomial ideals that include, as special cases, the ideals of 2-minors of ladders and the join–meet ideals of distributive lattices.

Let $S = K[x_a : a \in V(P)]$ be the polynomial ring over a field K, where $V(P)$ denotes the vertex set of P, and set $K[P] = S/I_P$. The classification of prime and radical polyomino ideals remains an open problem of particular interest. There are also deep connections between the algebraic properties of $K[P]$ and certain combinatorial constructions involving rook placements on the cells of P. This connection was first observed by Ene, Herzog, Qureshi, and Romeo, who showed that the Castelnuovo–Mumford regularity of $K[P]$ coincides with the rook number of P when P is an L-convex polyomino. Later, Qureshi, Rinaldo, and Romeo conjectured that the switching rook polynomial, which counts equivalence classes of non-attacking rook placements, coincides with the h-polynomial of $K[P]$.

In this talk, we discuss these ideas and present results identifying classes of polyominoes for which the above conjecture holds. We also introduce a refinement of rook polynomials through the notion of domino-stability, a combinatorial condition that explains when the switching rook polynomial is palindromic, corresponding to the algebra $K[P]$ being Gorenstein.

MSC 2010: 05A15, 05B50, 05E40, 68W30

Keywords: Polyomino, binomial ideal, rook polynomial, Castelnuovo-Mumford regularity, Hilbert-Poincaré series

Acknowledgement: This work was partially supported by TÜBİTAK under Project No. 124F113.

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Algebraic Stories From Graphs: One-Value and Two-Value Magma Algebras

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The interplay between directed graphs and algebraic structures provides a fertile ground for investigating discrete mathematical properties through ring-theoretic tools. This talk explores the structural properties of two distinct classes of algebras induced by directed graphs with no multiple edges: one-value and two-value graph magma algebras. While both structures utilize a binary operation defined by the graph's edges, they differ in their treatment of non-edges-either mapping to a zero element (one-value) or to the second vertex in the pair (two-value).

In the first part of the talk, we examine the ring structure of one-value graph magma algebras. We focus on the Jacobson radical J, which is shown to be a nilpotent ideal with $J^2 = 0$. A key result is the decomposition of any semiperfect one-value algebra into a direct sum of a semisimple ring and a ring isomorphic to an algebra induced by a graph without isolated idempotent vertices. We also characterize these structures in terms of their singular ideals and identify conditions under which they become right Kasch rings.

The second part transitions to two-value graph magma algebras over finite graphs. These are characterized as finite-dimensional basic algebras with Jacobson radical square zero. We demonstrate how the underlying graph admits a block decomposition into complete and null subgraphs, which induces a natural ring decomposition via central idempotents. Finally, we discuss the construction of injective hulls for simple modules, illustrating how their dimensions and submodule structures are rigidly determined by the local configuration of adjacent blocks in the graph.

This is a joint work with Gülhan Mısra Bayer and Bülent Saraç. This study was supported by The Scientific and Technological Research Council of Türkiye (TÜBİTAK) under the Grant Number 122F105.

Finiteness conditions on essential extensions of simple modules

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In this talk we consider the following property of a Noetherian ring A:

($\diamond$) injective hulls of simple A-modules are locally Artinian

Interest in property ($\diamond$) was renewed by a question of P.F.Smith during the 10th Antalya Algebra Days in 2008 when he asked me whether some algebras had the above property. In this talk, we will revised what was known and the results achieved since then with a special focus on skew polynomial rings over affine Noetherian commutative rings and explain how a ring theoretical problem led to delicate issues having connections to dynamical systems. This talk is based on joint work with K. Brown and J. Matczuk.

MSC 2010: 16D50, 16P40, 16S35

Keywords: Injective module, Noetherian ring, simple module, skew polynomial ring

Acknowledgement: Partially supported by FCT UID/00144/2025, doi: 10.54499/UID/00144/2025

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Combinatorics of Codes on Weighted Projective Planes

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Weighted projective spaces provide a natural meeting point between algebraic geometry, combinatorics, and coding theory. In this talk, I will describe how these three viewpoints interact in the study of Reed-Muller type codes arising from weighted projective planes $\mathbb{P}(1, a, b)$ over finite fields.

The algebraic side is encoded by the vanishing ideal of the rational points of the weighted projective plane. By determining a universal Gröbner basis for this ideal, we obtain explicit descriptions of the dimensions of the associated evaluation codes.

The geometric side comes from viewing weighted projective planes as toric varieties. Homogeneous polynomials of weighted degree d correspond to lattice points of a polygon (triangle, in this case)

$$P_d = \{(x, y) \in \mathbb{R}^2 : ax + by \leq d, x, y \geq 0\},$$

allowing geometric information to be translated into coding-theoretic parameters.

The heart of the talk will focus on the combinatorial techniques used to study minimum distances. Through projective reductions of lattice polygons, footprint methods, and a delicate analysis of lattice-point configurations, the minimum-distance problem is transformed into an optimization problem on finite sets of lattice points. This leads to explicit formulas, sharp lower bounds, and several cases where the exact minimum distance can be determined, see, e.g., [1].

The goal of the talk is to illustrate how combinatorial arguments can drive the solution of algebraic problems and how toric geometry provides the bridge connecting these two worlds.

MSC 2020: 14M25, 11T71, 14G05, 94B27

Keywords: Toric code, weighted projective space, error-correcting code, Reed-Muller code

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Quantum symmetries of noncommutative algebras

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Symmetries of a space can be described by group actions, which in turn can be realised as group actions on the algebra of functions on that space. Actions of Hopf algebras on noncommutative algebras are viewed as "quantum" analogues of such symmetries, where the algebras represent functions on a noncommutative space. Classical results in representation theory ensure that group representations decompose into irreducibles; Hopf algebras with this property are called semisimple. In 2014, Etingof and Walton showed that if a semisimple Hopf algebra acts on a commutative integral domain, the action must factor through a group implying that no genuine quantum symmetry exists for commutative domains. In this talk, we will present a new construction of semisimple Hopf algebras that yields genuine quantum symmetries of noncommutative domains.

MSC 2010: 16T05, 16S35

Keywords: Kac-Paljutkin, semisimple Hopf algebras

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Neat Homomorphisms

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Neat subgroups of abelian groups can be generalized to modules in several different ways. My PhD thesis advisor Rafail Alizade has suggested to investigate proper classes of short exact sequences of modules related with neatness, complements and supplements, and we studied some related problems with Salahattin Özdemir and Zübeyir Türkoğlu in their PhD studies, and with my colleague Engin Büyükaşık; see [2], [3] and [4]. The relations between these and other related proper classes and associated homological objects have been further investigated by the group around Rafail Alizade; see [1]. We shall mention some of these relations that we shall use for our main concern in this talk: neat homomorphisms (which need not be monic or epic). This is another related but a quite natural concept that we investigate with Salahattin Özdemir (see [5]): a homomorphism $f : A \to B$ of modules is said to be a neat homomorphism if it has no proper extension in the injective envelope $E(A)$ of A. It has been introduced by Edgar E. Enochs for torsion-free covering modules over commutative domains and further investigated by James J. Bowe; see [6] and [7]. Neat homomorphisms are exactly the homomorphisms that preserve the essential monomorphisms in pushout diagrams as observed by Helmut Zöschinger in the case of abelian groups [8]. Our main results are the following. Over a commutative domain where every maximal ideal is invertible, if an epimorphism $A \to B$ of modules is neat and $\operatorname{Rad} A = 0$, then it must be an isomorphism. A homomorphism $f : A \to B$ of modules over a Dedekind domain is neat if and only if $\operatorname{Ker} f \subseteq \operatorname{Rad} A$ and $\operatorname{Im} f$ is closed in B (that is, $\operatorname{Im} f$ has no proper essential extension in B). For a ring R, we investigate which proper class defining properties are satisfied by the class of all short exact sequences of R-modules determined by neat epimorphisms. Unlike the class determined by neat monomorphisms, this class is not a proper class unless R is a semisimple ring.

MSC 2010: 18G25, 16D40, 16D50, 13F05

Keywords: Neat homomorphism, neat submodule, proper class, C-ring, Dedekind domain

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New Trends in the Study of Homological Properties

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The study of homological properties has traditionally been carried out by simply determining whether a given object satisfies a certain condition or not, much like classical switches, which had only two possible states: ON and OFF.

More recently, however, a new trend has emerged in which the focus has shifted from deciding whether an object satisfies a given condition to measuring how close an object M is to satisfying it.

The central idea underlying this approach is to associate with M a class of objects consisting of all those objects with respect to which M satisfies the condition defining the homological property under consideration. These classes are known as the domains of M, and their introduction has significantly increased researchers' interest in this area.

To date, most of the work on this topic has been developed from a ring-theoretic perspective, leaving aside a fundamental viewpoint, namely that of homological algebra.

In this talk, we aim to lay the foundations of a homological approach to this theory by introducing its basic notions and establishing its fundamental results. We will also present an application illustrating that the ideas and techniques arising from this framework can be effectively employed to prove highly nontrivial results entirely within the realm of homological algebra.

MSC 2010: 34B05, 34A08, 90C35

Keywords: Gorenstein-like modules, (pre)covers and (pre)envelopes, homological domains

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Abstracts of Participants' Talks

An Assessment of the Arithmetic Exceptionality of Generalized Chebyshev Polynomials of the Second Kind

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A polynomial $f \in \mathbf{Z}[\mathbf{x}]$ in n variables is called *arithmetically exceptional* if the induced map

$$f : \mathbf{F}_p^n \to \mathbf{F}_p^n$$

is a permutation for infinitely many primes p. By proving Schur's conjecture, Fried [2] completed the classification of univariate exceptional polynomials, establishing that they are compositions of linear polynomials, power maps and the classical Chebyshev polynomials $P_{A_1}^k$ of the first kind. On the other hand, the classification of multivariate exceptional polynomials is still an open problem. It is known that the generalized Chebyshev polynomials $P_{\mathfrak{g}}^l$ of the first kind, where $\mathfrak{g}$ is a semi-simple Lie algebra of rank n, provide exceptional polynomials in n variables for any prime $l > e + 1$ where e is the exponent of the Weyl group associated with the Lie algebra $\mathfrak{g}$. Classical Chebyshev polynomials $Q_{A_1}^k$ of the second kind are not exceptional although they provide permutations under some restrictive conditions [1].

In this talk, we focus on the generalized Chebyshev polynomials $Q_{A_2}^k$ of the second kind associated with the Lie algebra A_2. We show that the bivariate polynomials $Q_{A_2}^k$ are not arithmetically exceptional for any integer $k \geq 2$. We achieve this by studying the field norms of certain cyclotomic elements that admit a parametrization of finite fields.

MSC 2010: 11T06, 11T22, 11T55

Keywords: Chebyshev polynomials, Lie algebra, Weyl group, field norm, arithmetic exceptionality

Acknowledgement: This study was supported by The Scientific and Technological Research Council of Türkiye (TÜBİTAK) under Project No. 124F146.

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Relative max-projectivity

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There is an extensive literature on the modules and rings whose projectivity domains are minimal or restricted. P-poor modules are introduced in [7] as the ones having their projectivity domains as small as possible, i.e. consisting of semisimple modules only. Recently, many studies have been conducted concerning p-poor modules along with their generalizations and rings that have exactly two feasible domains of projectivity: semisimple modules and all modules ([4, 6, 7]).

The concept of max-projectivity first appeared in [1] and [2] for characterizing the rings whose injective modules are R-projective (resp. max-projective). These rings are completely characterized for some classes of rings including, commutative Noetherian rings, local rings and right hereditary right Noetherian rings. The max-projectivity of a module can be tested by being projective with respect to all short exact sequences ending with simple modules. Existence of this testing class has been a motivation in [3] to the study of the relative max-projectivivity.

In this talk, using the notion of relative max-projectivity, we first introduce the max-projectivity domain of a module. Such a domain includes the class of all modules whose maximal submodules are direct summands (this class denoted as $MDMod\text{-}R$). We call a module max-p-poor if its max-projectivity domain is exactly the class $MDMod\text{-}R$. We establish the existence of max-p-poor modules over any ring. Furthermore, we study commutative rings whose simple modules are max-projective or max-p-poor. In the remaining part of the talk, we determine the right noetherian rings for which all right modules are either projective or p-poor. Max-p-poor abelian groups are fully characterized and shown to coincide precisely with p-poor abelian groups.

MSC 2010: 16D50, 16D60, 18G25

Keywords: Max-projective modules, p-poor modules, max-p-poor modules.

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Square-Free and Dual-Square-Free Abelian Groups

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Summand-square-free and dual-square-free modules are important classes of modules that have been studied extensively in the literature. However, the structure of these modules has not yet been investigated sufficiently. In this talk, we will present some characterizations of these classes of modules over the ring of integers.

Theorem 1. *An abelian group G is square-free if and only if:*

(i) $rk_0(G/T(G)) = 1$*, and*

(ii) $G_p \cong \mathbb{Z}p^n$ *(for* $n \in \mathbb{Z}, n \geq 0$*) or* $\mathbb{Z}_{p^\infty}$ *for each* $p \in \Omega$*.*

Theorem 2. *If $T(G)$ and $G/T(G)$ are summand-square-free, then G is also summand-square-free.*

Summand-square-free and dual-square-free torsion and finitely generated abelian groups are completely characterized. If G is a summand-square-free group, then $T(G)$ is summand-square-free. We give an example of a summand-square-free group G such that $T(G)$ is summand-square-free but $G/T(G)$ is not.

Torsion-free dual-square-free groups have rank less than or equal to 2. Every rational group is dual-square-free. The class of dual-square-free groups is not closed under extensions.

MSC 2010: 16D70, 16L30, 20K25, 20K40

Keywords: Square-free modules, summand-square-free modules, dual-square-free modules

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(p, q)-Stirling Numbers of Type B via Signed Restricted Growth Functions and Rook Theory

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In this talk, we present some results from our recent work [2] on (p, q)-Stirling numbers of type B from a rook-theoretic point of view. The study is motivated by the classical (p, q)-Stirling numbers introduced by Wachs and White [4], the type B analogue of restricted growth functions [1], and rook theory on Ferrers boards [3].

We introduce a type B Ferrers board and establish a bijection between signed restricted growth functions and rook placements on this board. This correspondence allows us to define some combinatorial statistics on signed restricted growth functions and rook placements, whose joint distribution gives the (p, q)-Stirling polynomials of type B. We also present recurrence relations and generating functions for these polynomials. Finally, we introduce type B Laguerre boards and show that their rook numbers coincide with the Lah numbers of type B.

MSC 2010: 05A05, 05A15, 05A18, 11B73

Keywords: (p, q)-Stirling numbers, Lah numbers, restricted growth functions, rook placements, Ferrers boards, Laguerre boards

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Comparable Overrings of Marot Rings

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The study of overrings and their comparability properties play a significant role in commutative ring theory. Recently, Ahmed Ayache characterized comparable overrings of integral domains. In this talk, we generalize this question to Marot rings. A Marot ring is a commutative ring with zero divisors where every regular ideal is generated by its regular elements.

To explore the comparable overrings of Marot rings, we construct structures similar to pseudo-valuation domains and divided domains. Specifically, we introduce new concepts such as r-divided prime ideals and r-pseudo-valuation rings (r-PVR). Our main results characterize a comparable overring R_o of a Marot ring R, especially when $\overline{R} \subset R_o$. We prove that such an R_o is a proper valuation overring of R, and its unique regular maximal ideal is an r-divided prime ideal of R. Furthermore, we provide a complete characterization for Marot Prüfer rings, showing that their comparable overrings are exactly the regular localizations $R_{(m_o)}$ at r-divided prime ideals m_o.

MSC 2010: 13A15, 13F30, 13B30, 13B22

Keywords: Marot rings, valuation rings, comparable overrings, regular localization, r-pseudo-valuation rings (r-PVR)

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$\oplus$-Cofinitely Radical Supplemented Lattices

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In this talk, $\oplus$-cofinitely radical supplemented lattices are introduced and studied. It is shown that for a lattice L satisfying $r(L) \ll L$, the notions of $\oplus$-cofinitely Rad-supplemented and $\oplus$-cofinitely supplemented lattices coincide. For lattices with the Summand Sum Property (SSP), it is proved that if $1 = \bigvee_{i \in I} a_i$, where each a_i is a direct summand of L and each $a_i/0$ is $\oplus$-cofinitely Rad-supplemented, then L is also $\oplus$-cofinitely Rad-supplemented. Moreover, a characterization is obtained in terms of w-local direct summands: a lattice L with SSP is $\oplus$-cofinitely Rad-supplemented if and only if $1/\mathrm{wLoc}^{\oplus}(L)$ has no maximal element. It is further shown that every cofinite element of $1/r(L)$ is a direct summand of $1/r(L)$ whenever L is $\oplus$-cofinitely Rad-supplemented, while the converse does not hold in general. In addition, quotient sublattice properties related to fully invariant elements are established.

MSC 2020: 06C05, 06C15

Keywords: Lattices, small elements, supplemented lattices, radical supplemented lattices, direct summands, w-local elements.

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On Semigroups of Order-Preserving Transformations with the Same Fix Set

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Let $\mathcal{O}_n$ be the semigroup of all order-preserving (full) transformations on the finite chain $X_n = \{1, \ldots, n\}$ under its natural order. In this talk, for a singular idempotent ξ, we show that

$$\mathcal{O}_n(\xi) = \{\alpha \in \mathcal{O}_n : \alpha^m = \xi \text{ for some } m \in \mathbb{N}\}$$

is a maximal nilpotent subsemigroup of $\mathcal{O}_n$ with zero ξ. Moreover, for a non-empty subset Y of X_n, we give a necessary and sufficient condition for the set $\mathcal{O}_n(Y)$ to be a subsemigroup. Then we find a unique minimal generating set, and so rank, of $\mathcal{O}_n(Y)$ whenever it is a subsemigroup of $\mathcal{O}_n$. Every subset Y of X_n such that $\mathcal{O}_n(Y)$ is (completely) isolated is characterized.

MSC 2020: 20M20, 20M05

Keywords: Order-preserving transformation, orientation-preserving, (completely) isolated subsemigroup, generating set, rank

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Variants of the Szemerédi, Green–Tao, and Sárközy Theorems for Harmonic Progressions

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The celebrated theorems of Szemerédi and Green–Tao establish the existence of arbitrarily long arithmetic progressions in subsets of the integers with positive density and within the prime numbers, respectively. In this talk, we establish the existence of arbitrarily long harmonic progressions within specific subsets of the positive integers. Specifically, we obtain a harmonic analog of Szemerédi's theorem for multiplicatively closed sets and establish shifted versions of both the Szemerédi and Green–Tao theorems for harmonic progressions. Additionally, we present a Sárközy-type result within this framework. These results illustrate how arithmetic regularity properties translate into harmonic contexts.

MSC 2010: 05D10, 11B25, 11N13

Keywords: Arithmetic progressions, harmonic progressions, Sárközy-type theorems

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Some Graph Dimensions over special Monoid Presentations

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The main target of this talk is to partially address the open problem of "characterizing all graphs having infinite multiset dimensions" through graphs obtained from special minimal (efficient or inefficient) monoid presentations. If there is time left in the talk, we will also reveal the relationships between metrics, multiset, outer multiset dimensions, and special generating functions.

MSC 2010: 05C35, 05C80, 11B68, 12D10, 20F05, 57M15

Keywords: Minimal presentation, graphs, metric dimension, multiset dimension, outer multiset dimension, local multiset dimension

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Torsion Obstruction for Conclusive Posets

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Derivations of incidence algebras of posets lie at an intersection of combinatorics, algebra and topology and problems in this setting often admit interpretations in each of these languages. A central example is the existence of outer derivations of an incidence algebra, which is equivalent to the non-vanishing of its first Hocschild cohomology group. From a topological perspective, this corresponds to the non-vanishing of the simplicial cohomology of the order complex of the poset.

The computational complexity of detecting outer derivations can depend strongly on the choice of codomain of the derivations. In particular, one may ask whether the existence of outer derivations is independent of the chosen codomain, in the sense that they either exist for every non-trivial choice of codomain or for none. If this independence holds, then computations can be simplified by performing the search in the field of characteristic 2. Posets satisfying this property are called conclusive. In [3], authors study this phenomenon and conjecture that all posets are conclusive.

We provide a counterexample to this conjecture by showing that it fails for the minimal finite model of the real projective plane $\mathbb{R}P^2$, due to the presence of torsion in homology. In order to preserve the combinatorial spirit, we will provide a proof that does not rely on heavy topological machinery. We will also obtain a purely combinatorial necessary and sufficient condition for conclusiveness.

MSC 2010: 06A11, 16E40

Keywords: Conclusive poset, derivation, minimal finite model, real projective plane, torsion

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$SL(5)$ Exceptional Drinfel'd Algebroids

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T-duality in string theory can be understood in terms of different Manin triple decompositions of the Drinfel'd double associated with a Lie bialgebra. Recently, $SL(5)$ exceptional Drinfel'd algebras have been introduced in the physics literature as an analogous algebraic structure underlying U-duality. In this talk, we take a closer and more mathematically rigorous look at these algebras and extend them to the algebroid setting. To achieve this, we employ the algebroid calculus framework that we developed together with Nambu-Poisson structures, exceptional generalized geometry, and formal bundle rackoids.

MSC 2010: 53Z05, 81T30, 53A35

Keywords: Dualities, algebroids, calculus on algebroids, exceptional generalized geometry

Unique decomposition into w-Ideals for strong Mori domains

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A commutative ring R has the unique decomposition into ideals (UDI) property if, for any R-module that decomposes into a finite direct sum of indecomposable ideals, this decomposition is unique up to the order and isomorphism classes of the indecomposable ideals. In [2], the UDI property has been characterized for Noetherian integral domains. In this talk, we present the UDI-like property (UDwI property) for strong Mori domains; domains satisfying the ascending chain condition on w-ideals.

MSC 2010: 13A15, 13F05, 13C05, 20K15

Keywords: UDI property, the w-operation, strong Mori domain

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Cohen–Macaulay and Gorenstein Properties of Bi–Amalgamated Algebras with Applications to Algebroid Curves

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The amalgamated duplication (or the gluing) of a ring with one of its ideals has been inroduced by D'Anna in [2] as a tool of constructing Gorenstein algebroid curves, which are of special interest in the theory of curve singularities. The study of Cohen–Macaulay and Gorenstein properties were extended to the more general amalgation construction in [3], albeit with no particular applications to algebroid curves. In this talk, we characterize the Cohen–Macaulay and Gorensteinness of the bi–amalgamated algebra and present applications of our results to algebroid curves [1]. We first calculate the depth and dimension of the bi–amalgamated algebra using their cohomological interpretation and characterize Cohen–Macaulayness as a result. Assuming Gorensteinness, we give a construction of the canonical module of the underlying ring. Under some additional assumptions, we also completely determine when the bi–amalgamation is Gorenstein by utilizing the pull–back description. Finally, we prove that amalgamations and bi–amalgamations of algebroid curves also give rise to algebroid curves. This gives us a process of starting with an arbitrary algebroid curve and then gluing it with certain ideals in order to obtain a Gorenstein one.

MSC 2010: 13H10, 13B02, 13C15, 14H20

Keywords: Bi–amalgamated algebra, amalgamated algebra, Cohen–Macaulay ring, Gorenstein ring, algebroid curve

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Kolmogorov Superposition Theorem and Kolmogorov-Arnold Networks

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In 1956, V. Arnold based on A. Kolmogorov's ideas gave the affirmative answer to the question: can every continuous function of three variables be expressed as a composition of finitely many continuous functions of two variables? (Hilbert's thirteenth problem proposed in 1899), and a bit later, in 1957, Kolmogorov (see [1]) simplified Arnold's method and proved a remarkable (philosophical, in our opinion) result (known as the Kolmogorov-Arnold representation theorem) for continuous functions: any continuous real function of several variables defined on an n-dimensional cube can be represented as a superposition of continuous functions of one variable and a single function of two variables.

This talk provides a geometric explanation for this remarkable result in case of two variables and then demonstrates how this classic discovery in pure mathematics has found application more than 60 years later in modern deep learning (Kolmogorov-Arnold networks, or KANs) (see [2], [3]). KANs use the main idea of the theorem to create a trainable setup. They swap the standard neural network's fixed activation functions with trainable splines. This leads to models that are easier to understand and work well in accuracy. Plus, they stand up better to something called "catastrophic forgetting." Although, KANs face their own set of issues they are potential competitors to multilayer perceptrons (MLPs) in scientific computing and point to new directions for future research.

MSC 2010: 26B40, 92B20, 82C32, **MSC 2020:** 68T07

Keywords: Hilbert's thirteenth problem, Kolmogorov–Arnold superposition theorem, Kolmogorov–Arnold Networks, neural networks

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On Normal Projective Hypermodules

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Let R be a Krasner hyperring and M be a left Krasner R-hypermodule. M is called normal projective if for every surjective normal homomorphism $g : N \to P$ and every normal homomorphism $f : M \to P$, there exists a normal homomorphism $h : M \to N$ such that $g \circ h = f$ [8]. In this talk, we give some characterizations of normal projective hypermodules. Moreover, we provide illustrative examples.

MSC 2020: 20N20, 18E10, 16D90

Keywords: Normal projective hypermodule, free hypermodule

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Boosting the Partial Sum Attack on AES

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Block ciphers are fundamental building blocks of modern cryptography and serve as critical components in a wide range of cryptographic constructions, including message authentication codes and hash functions. Among them, the Advanced Encryption Standard (AES), standardized by NIST, is one of the most widely used symmetric encryption algorithms. It is extensively deployed in protocols such as TLS, GSM-5G, and WPA, as well as in applications including file encryption and secure messaging systems (e.g., WhatsApp). The core components of the AES algorithm are based on arithmetic over the finite field $\mathbb{F}_{256}$ and functions defined on this field.

The partial-sum technique, which exploits the characteristic-2 structure of $\mathbb{F}_{256}$, has been regarded for more than two decades as the fastest known method for attacks on 6-round AES. A recently proposed FFT-based improvement has further accelerated this approach. In this work, we present a new cryptanalytic technique that achieves an approximate 2^8-fold speedup over both the classical and FFT-based partial-sum attacks. In this respect, the proposed method constitutes the fastest known key-recovery attack on 6-round AES to date.

MSC 2020: 12E20, 94A60, 94B05

Keywords: AES (Advanced Encryption Standard), finite fields, MDS matrices, the partial-sum technique, encryption algorithms

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Some Results on Quantum MDS Codes

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In this talk, we present some recent results on quantum error-correcting codes. After a brief introduction to the basic principles of quantum error correction, we focus on the connection between classical maximum distance separable (MDS) codes and quantum MDS (QMDS) codes through Hermitian self-orthogonality. In particular, we discuss constructions arising from generalized Reed–Solomon codes and related algebraic methods over finite fields. The main goal of the talk is to explain how suitable families of classical MDS codes can be used to derive quantum codes with strong parameters, and to highlight the conditions under which these constructions produce QMDS codes.

Keywords: Quantum error correction, quantum MDS codes, Hermitian self-orthogonality, generalized Reed–Solomon codes, finite fields

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On Thompson's Critical Subgroup

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Let p be a prime and let P be a finite p-group. In this talk, we introduce a new characteristic subgroup $K(P)$ of P and show that it is a critical subgroup in the sense of Thompson. We also define further characteristic subgroups $K^*(P)$ and $K_i(P)$, for each positive integer i, and discuss several structural properties of these subgroups which are analogous to those of critical subgroups.

As an application, for a finite solvable group G, we define a new characteristic subgroup $Y(G)$ and prove that it satisfies an analogue of Thompson's theorem for solvable groups.

MSC 2010: 20D10, 20D15, 20D45

Keywords: p-groups, critical subgroup, characteristic subgroup, Thompson's theorem, solvable groups

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Counting Arithmetic Progressions in Finite Rings

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Over the last hundred years, one of the major problems in mathematics has been to find arbitrarily long arithmetic progressions inside special subsets of the integers. A famous result in this area was proved by Endre Szemerédi in 1975. His theorem says that every subset of the positive integers with positive upper density contains arithmetic progressions of any length. In this talk, we will discuss results about counting 3-term arithmetic progressions in certain subsets of finite rings.

MSC 2020: 11B25, 11T24, 11T30

Keywords: Arithmetic progressions, Roth's theorem, finite rings

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Duality in Derived Category of $\mathcal{O}^\infty$

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Let $\mathfrak{g}$ be a reductive Lie algebra over a p-adic field F with a split Cartan algebra $\mathfrak{t}$ and a Borel subalgebra $\mathfrak{b} \supset \mathfrak{t}$. In analogy with the classical category $\mathcal{O}$ of Bernstein-Gelfand-Gelfand, we define category $\mathcal{O}$ for $(\mathfrak{g}, \mathfrak{b}, \mathfrak{t})$, and the thick category $\mathcal{O}^\infty$, which is the smallest abelian subcategory of the category of all $\mathfrak{g}$-modules which contains $\mathcal{O}$ and is stable under extensions. We show that the functor $\mathrm{RHom}_{U(\mathfrak{g})}(-, U(\mathfrak{g}))$ preserves $D^b_{\mathcal{O}^\infty}(U(\mathfrak{g}))$, which is the subcategory of complexes of $U(\mathfrak{g})$-modules with cohomology modules in $\mathcal{O}^\infty$. From a result of Coulembier-Mazorchuk, we deduce that this subcategory is equivalent to $D^b(\mathcal{O}^\infty)$. We then introduce the duality functor $\mathbb{D}^{\mathfrak{g}}$ on $D^b(\mathcal{O}^\infty)$. Finally, we explain how these constructions are related to locally analytic representations.

MSC 2010: 22E50, 20G05, 11E95

Keywords: p-adic Lie groups, Lie algebras, derived categories

On re-Small Submodules

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In this note, we introduce the notion of an re-small submodule as a generalization of r-small submodules. A submodule V of a module M is said to be *re-small* if $V \ll \mathrm{Rad}(E(V))$, equivalently $V \ll \mathrm{Rad}(E(M))$, where $E(V)$ denotes the injective hull of V. This notion is denoted by $V \ll_{re} M$. It is well known that a module M is called *cosingular* if the submodule $\mathcal{Z}^*(M) = M \cap \mathrm{Rad}(E(M))$ equals M [2]. We show that every small submodule of a cosingular module is re-small. We also prove that the class of re-small modules is closed under submodules and finite sums. Moreover, over left hereditary rings, the class of re-small modules is closed under factor modules and extensions. In addition, we introduce the notion of re-supplemented modules and study several of their properties.

MSC 2010: 16D10, 16D60

Keywords: Small submodules, radical, injective hull, re-small submodules, re-supplemented modules

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Proper Classes Generated by Isosimple Modules

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The proper class projectively generated by isosimple right modules and the one injectively generated by isosimple right modules do not coincide in general, even for abelian groups. Commutative principal ideal domains in which these two proper classes coincide are fields; and the rings all of whose right modules are coprojective with respect to the latter proper class are right strongly V-rings (i.e., the rings all of whose isosimple right modules are injective).

MSC 2010: 18G25, 16D40, 16D50, 16D60

Keywords: (I-) neat submodule, (I-) coneat submodule, coprojective module, isosimple module, right (strongly) V-ring.

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$\ast$-(Strongly) $\mathcal{G}$-Clean Rings with Involution

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In this talk, we introduce the notions of $\ast$-$\mathcal{G}$-clean and $\ast$-(strongly) $\mathcal{G}$-clean rings. We investigate the relationship between these classes and the corresponding notions of $\mathcal{G}$-clean and strongly $\mathcal{G}$-clean rings, and provide conditions under which the converse implications hold. Several fundamental properties of $\ast$-(strongly) $\mathcal{G}$-clean rings are obtained, including their behavior under homomorphic images, factor rings, formal power series rings and direct products. Furthermore, we derive Jacobson radical characterizations of $\ast$-(strongly) $\mathcal{G}$-clean rings.

MSC 2010: 16L30, 16S50, 16U60

Keywords: $\mathcal{G}$-idempotent element, $\mathcal{G}$-clean ring, $\ast$-(stronly) $\mathcal{G}$-clean ring, $\ast$-$\mathcal{G}$ exchange ring

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Direct Products in the Category of Hypermodules

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Let R be a hyperring. We denote by $\mathbf{HypMod}(R)$ the category whose objects are all R-hypermodules and whose morphisms are all normal homomorphisms. This presentation focuses on the direct product and the external direct sum of hypermodules. In particular, we prove that $\mathbf{HypMod}(R)$ admits products (and pullbacks), but does not admit coproducts (and pushouts). Furthermore, using the direct product of two given hypermodules M and N, we prove that the set $\mathrm{Hom}_n(M, N)$ of all normal homomorphisms from M to N does not admit a canonical hypergroup, in general. This also shows that $\mathbf{HypMod}(R)$ is not an abelian category.

MSC 2020: 20N20, 18A30, 16Y99

Keywords: Krasner hypermodules, the direct product, product, pullback

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The Proper Class Projectively Generated by Coatomic Modules

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Let R be a ring. We denote by CA the class of all coatomic R-modules. In this talk, we will present results on the objects of the proper class CA-$Pure$ projectively generated by CA. It is shown that; (1) CA-$Pure = Split$ if and only if R is a left Artinian serial ring with $\text{Rad}^2(R) = 0$; (2) $Neat = CA$-$Pure$ if and only if R is a semiartinian ring; (3) A coatomic module M is $CA - flat$ if and only if it is projective.

MSC 2020: 16D40, 16D50, 16E10

Keywords: CA-$Pure$, CA-flat, pure submodule, CA-pure submodules

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Algebras Determined by the Action of Bilinear Maps on Pairs of Inverses

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Motivated by the study of rational functional identities, we introduce a class of algebras in which the global structure of a bilinear map is determined by its behavior on pairs of inverses. We establish a decomposition of bilinear maps that vanish on such pairs. We then study basic properties of such algebras, providing a characterization in terms of Jordan derivations alongside various key examples. We also examine their relation to zero Lie product determined algebras. Finally, we apply this framework to characterize certain linear preservers and to solve a comprehensive rational functional identity on finite-dimensional simple algebras.

MSC 2010: 16R60, 16U60, 16W25

Keywords: Unity product determined algebra, Rational functional identity, Jordan derivation, Zero Lie product determined algebra, Unity product preserving map

Acknowledgement: The first author acknowledges the grant from the Scientific and Technological Research Council of Türkiye (TÜBİTAK) under the 2211-E National PhD Scholarship Program for Former Undergraduate and MSc/MA Scholars.

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Harmonic Numbers and Their Computational Complexity Analysis

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This talk surveys the computational landscape of harmonic numbers. We will begin by introducing several optimized algorithms alongside related heuristic results concerning $J(p)$ sets. Next, we will review key literature focused on the computational aspects of harmonic numbers, systematically evaluating and comparing the efficiency of different computation methods. Finally, we will explore various generalizations of harmonic numbers, analyzing their respective algorithms through the lens of computational complexity theory.

MSC 2010: 11Y16, 68Q25, 11B75

Keywords: Harmonic numbers, algorithmic number theory, complexity Analysis, computational optimization

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On RD-Projectively Poor Modules

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The notion of poor modules, introduced by Alahmadi, Alkan and López-Permouth in [1], initiated the study of modules whose injectivity domains are as small as possible. Motivated by this perspective, several projectivity-type analogues have been developed, including projectively poor (shortly p-poor) and pure-projectively poor (briefly pp-poor) modules introduced in [2].

On the other hand, Warfield introduced the notion of RD-exact sequences in [3]. A short exact sequence

$$0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0$$

is called RD-*exact* if for every cyclically presented module M, the sequence $\mathrm{Hom}(M,B) \longrightarrow \mathrm{Hom}(M,C) \longrightarrow 0$ is exact. This framework gives rise to the notions of RD-projective and RD-injective modules and has become an important tool in the study of various classes of rings and modules.

The notion of RD-projectively poor (shortly, RDp-poor) modules was introduced by Yiğit and Toksoy in [4] as an RD-theoretic analogue of p-poor and pp-poor modules. A module is said to be RDp-poor if its RD-projectivity domain consists only of RD-split modules.

In [4], RDp-poor modules and their relationships with p-poor and pp-poor modules were studied. Various conditions ensuring the coincidence of these notions were established, the significance of RD-split modules was examined and rings over which every module is RDp-poor were characterized. In this talk, some of the main results of that work will be presented.

MSC 2010: 16D10, 16D80

Keywords: Poor module, projectively poor module, pure-projectively poor module, RD-projectively poor modules, projectivity domain

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Abstracts of Poster Presentations

On the Equality of the Fuzzy Jacobson Radical and Fuzzy Nilradical for Fuzzy Ideals of Artinian Rings

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In this study, the radical structures of fuzzy ideals defined on a commutative ring R with identity are investigated. Specifically, the well-defined concepts of the fuzzy Jacobson radical ($J(\mu)$) and the fuzzy nilradical ($Nil(\mu)$) of a given fuzzy ideal μ, introduced by J.N. Mordeson and D.S. Malik, are brought into focus. The necessary algebraic thresholds for extending the fundamental classical equality of the Jacobson radical and nilradical in Artinian rings to these fuzzy substructures are examined. The framework is based on the characterization that a ring R is Artinian if and only if any fuzzy ideal of R has a finite image and its alpha-cuts satisfy the descending chain condition (DCC). Building upon this structural foundation, the absolute inequality boundaries between the membership degree of the ring's identity element, $\mu(1_R)$, and the membership degrees of the nilpotent elements are proved in order to fully satisfy the fuzzy equivalence $J(\mu) = Nil(\mu)$. The findings reveal the fuzzy space counterparts of the stopping points of classical ideal chains.

MSC 2010: 13E10, 03E72, 13A15

Keywords: Fuzzy ideal, Jacobson radical, α-cut, Artinian ring, nilradical

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Algebraic Properties of Higher-Order Complex Differential Operators

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Higher-order complex differential operators arise naturally in the study of generalized analytic functions, boundary value problems, and complex partial differential equations. Besides their analytical significance, these operators possess rich algebraic structures that play an important role in understanding the solvability and representation of solutions of associated differential equations. In this work, we investigate several algebraic properties of higher-order complex differential operators and discuss their applications in complex analysis.

The study focuses on operators generated by Wirtinger derivatives and their higher-order compositions. Particular attention is devoted to the algebraic behavior of these operators under addition, composition, and iteration. The kernel and image spaces associated with higher-order operators are analyzed, and their relations with generalized analytic functions are examined. Furthermore, decomposition techniques are employed to represent higher-order equations through systems of lower-order equations, providing a useful algebraic framework for studying their solution spaces.

The obtained results demonstrate that several classes of higher-order complex differential equations admit representations revealing underlying algebraic structures. These representations facilitate the investigation of existence and uniqueness questions and provide insight into the interaction between operator theory and complex analysis. In addition, the presented approach establishes connections between differential operators and linear algebraic methods frequently used in the analysis of functional spaces.

The proposed framework contributes to the growing interaction between algebra and complex analysis and offers a basis for future studies involving generalized analytic functions, operator algebras and higher-order boundary value problems.

MSC 2010: 30G20, 35F45, 47A15

Keywords: Higher-order complex differential operators, generalized analytic functions, operator theory, algebraic structures, complex analysis

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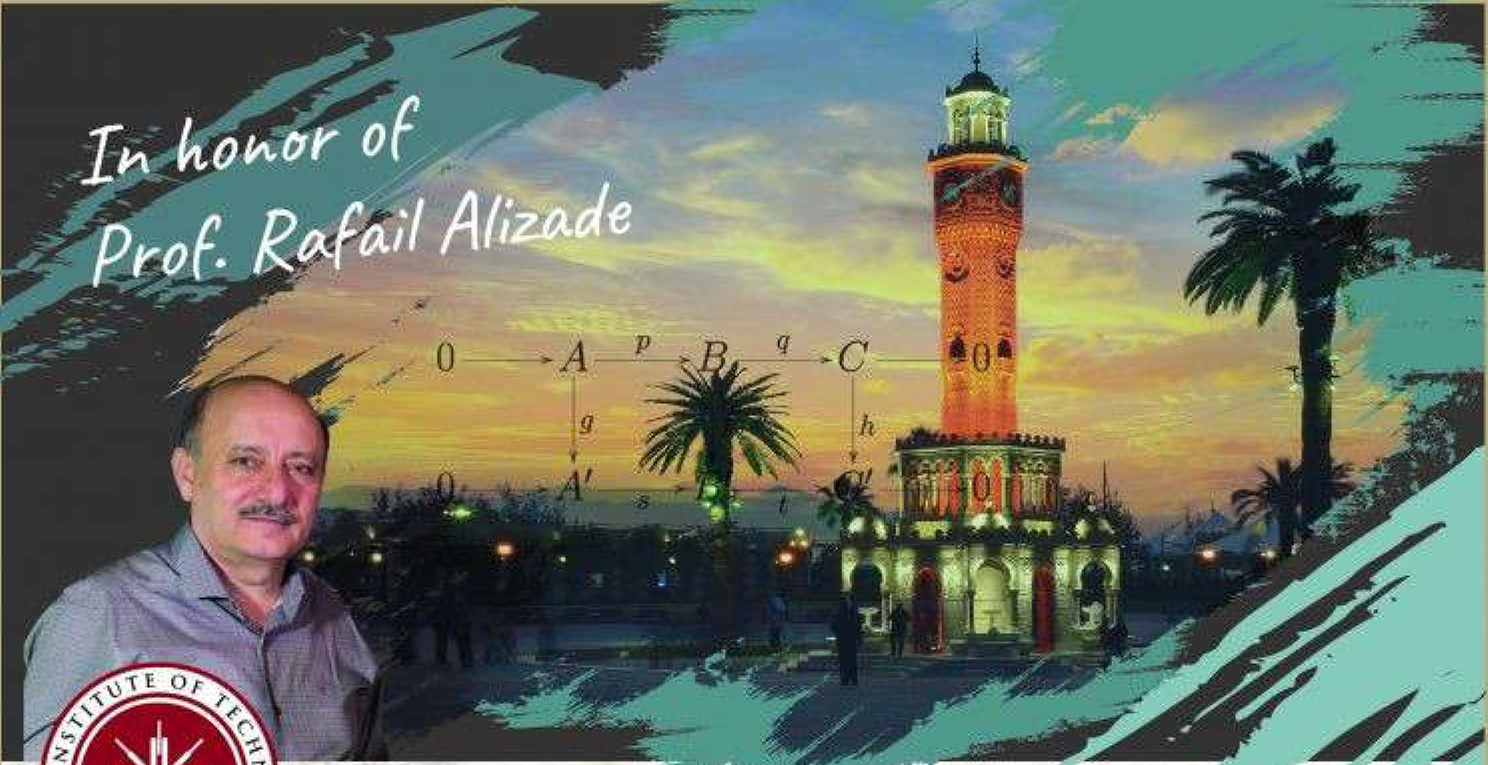

II. ANATOLIAN ALGEBRA WORKSHOP

July 8-10, 2026

İZMİR INSTITUTE OF TECHNOLOGY

Invited Speakers

Fatma Altunbulak Aksu
Mimar Sinan G.S.U.

Murat Altunbulak
Dokuz Eylül University

Pınar Aydoğdu
Hacettepe University

Paula Carvalho
Porto University

Christian Lomp
Porto University

Engin Mermut
Dokuz Eylül University

Luis Oyonarte Alcala
Almeria University

Mesut Şahin
Hacettepe University

Ayesha Asloob Qureshi
Sabancı University

Umut Varolgüneş
Koç University

TÜBİTAK

TÜRK MATEMATİK DERNEĞİ